

# Testing the Tail Index in Autoregressive Models

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## *Abstract*

The talk is based on a joint work with Hira L. Koul (*Michigan State University*) and Jan Picek (*Technical University in Liberec*).

The study of the extremal events such as the extreme intensity of the wind, the high flood levels of the rivers or extreme values of environmental indicators, or maximal or minimal performance of a portfolio naturally lead one to study the tails of the underlying distribution rather than its central part. Classical goodness-of-fit tests for a distribution are usually concerned with the central part, hence they cannot provide a sufficient information on the shape of its tails. Our primary goal is to decide whether the underlying distribution function is light- or heavy-tailed. The problem is semiparametric in nature, involving an unknown slowly varying function, besides the real-valued parameters of interest. A decision in favor of a heavy tail distribution would then suggest to study the shape of the tail more closely.

Testing the hypothesis on the tail index of a heavy tailed distribution is an alternative inference to the classical point estimation, surprisingly not yet much elaborated in the literature. Jurečková and Picek (2001) constructed the nonparametric tests for the sequence of *i.i.d.* observations. We construct a class of tests on the tail index of the innovation distribution in a stationary linear autoregressive model. The tests are nonparametric and are based on the series of residuals with respect to an appropriate estimator of the AR parameters; more precisely, they are based on the empirical process of maximal residuals of non-overlapping segments of such series. The simulation study illustrates a very good level performance of the tests. Such tests would find many applications in the environmental, financial and other time series. Similar technique can be used also for time series of other types.

## **References**

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