Published in Electronics Letters 51, issue 22, 1767-1769, 2015
which should be used for any reference to this work

Stability limitations from optical detection in Ramsey-type vapour-cell atomic clocks

S. Kang, M. Gharavipour, C. Affolderbach and G. Mileti

In today's state of the art compact vapour-cell atomic clocks relying on the pulsed Ramsey-type interrogation, optical detection noise is a major limitation to the achievable short-term stability. In this communication the influence of the optical detection time on the clock's short-term stability is investigated and a new analytical expression is developed to precisely predict the stability performance, taking into account the details of the optical detection phase of a Ramsey-type atomic clock. The theory is in good agreement with the experimental results. It is applied for evaluating the clock's shot-noise limit.

Introduction: Advanced compact vapour-cell atomic clocks operated in pulsed or periodic mode based on the Ramsey technique, e.g. pulsed optically pumped (POP) Rb clocks [1, 2] or Ramsey coherent population trapping (CPT) clocks [3], have realized short-term frequency stabilities of ~1-3×10^{-13} $\tau^{-1/2}$. For such clocks using the transmitted light detection, the optical detection noise is either the dominant or a non-negligible factor limiting the clock's short-term stability. Instability contributions arising from the local oscillator (LO) phase noise via the Dick effect have been comprehensively analysed using different approaches [4-8]. But only few articles have detailed theoretically on the instability contribution induced by the optical detection noise [1, 3]. In this communication, we use the geometrical approach utilized in analysis of the Dick effect to study how the detection noise contributes to a Ramsey clock's instability and develop an analytical formula to evaluate its performances.

Theoretical Description: The standard operation sequence for a Ramsey-type atomic clock based on POP technique with optical detection is shown in Fig.1. After a "strong" (~10 mW) optical pumping pulse (duration T_p) for state preparation, atoms experience two identical and successive $\pi/2$ microwave pulses (duration T_1) separated by a free evolution Ramsey time (T_{Ramsey}). At the end of the cycle, a "weak" (~100 µW) detection light pulse (duration T_d) reads out the atomic response and its amplitude information is acquired by the clock loop during a detection time τ_d (normally $\tau_d=T_d$). Similarly to the (microwave) phase-noise down-conversion in the Dick effect [1,2], the periodic optical detection also causes down-conversion of out-of-band amplitude modulation (AM) noise, near the cycle operation frequency $f_c = 1/T_c$ (T_c is the cycle time) and its harmonics, into the clock's loop, thus serving the LO with a false information.

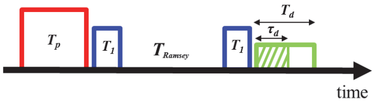


Fig. 1 *Pulse operation sequence for a conventional Ramsey-type atomic clock (cycle time* $T_c=T_p+T_1+T_{Ramsey}+T_1+T_d$*).*

In order to investigate the optical detection noise's influence on the clock's stability, we write the sensitivity function to the optically detected atomic signal as

$$g(t)=\begin{cases}0, & 0\le t<T_c-\tau_d\\ 1, & T_c-\tau_d\le t\le T_c\end{cases}\tag{1}$$

and its Fourier coefficients (g_n^c and g_n^s) at nth harmonic of f_c for cos and sin components and the average value (g_0) are given by:

$$g_n^c=\frac{1}{T_c}\int_0^{T_c} g(t)\cos\left(\frac{2\pi nt}{T_c}\right)dt, n\, is\, an\, integer\tag{2}$$

$$g_n^s=\frac{1}{T_c}\int_0^{T_c} g(t)\sin\left(\frac{2\pi nt}{T_c}\right)dt, n\, is\, an\, integer\tag{3}$$

$$g_0=\frac{1}{T_c}\int_0^{T_c} g(t)dt.\tag{4}$$

For very low Fourier frequencies, such a down-converted detection noise can be assumed white and will be completely compensated in locked frequency loop [4, 5]. So the final white AM clock loop noise $S_{RIN}^{LLO}(0)$ in the narrow range (at frequencies both above and below harmonics nf_c) is,

$$S_{RIN}^{LLO}(0)=2\sum_{n=1}^{\infty}\frac{(g_n^s)^2+(g_n^c)^2}{g_0^2}S_{RIN}^{\det}(nf_c)\tag{5}$$

where $S_{RIN}^{det}(f)$ is the power spectral density of the relative intensity noise (RIN) for the optical detection signal. For a given detection time τ_d, the corresponding white AM noise variance σ_{RIN}^2 and the Ramsey-type clock's signal to noise ratio (SNR) can be expressed as

$$\sigma_{RIN}^2=\frac{S_{RIN}^{LLO}(0)}{2\tau_d}\tag{6}$$

$$SNR=\frac{C}{\sigma_{RIN}}\tag{7}$$

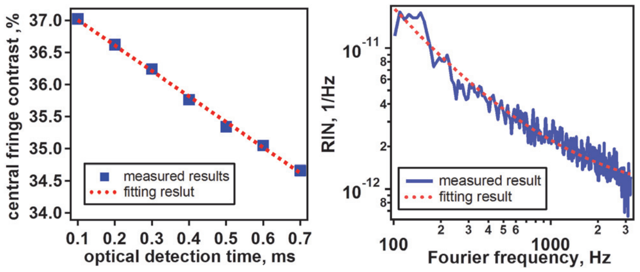
where C is the central fringe contrast of the Ramsey pattern. Following the well-known periodic operation clock stability formula [9] and using (5) to (7), the final clock instability induced by the optical detection noise can be expressed as

$$\sigma_y^{\det}(\tau)=\frac{1}{\pi}\frac{1}{C}\frac{1}{Q_a}\left(\sum_{n=1}^{\infty}\mathrm{sinc}^2(\pi nf_c\tau_d)S_{RIN}^{\det}(nf_c)\right)^{-1/2}\sqrt{\frac{T_c}{\tau_d}}\tau^{-1/2}\tag{8}$$

where $Q_a=\nu_0\cdot 2\cdot T_{Ramsey}$ is the atomic quality factor of the clock signal. The analytical formula (8) shows that the clock instability induced by the detection noise significantly depends on the detection time, which has not been explicitly presented in previous work [1, 3].

Experimental Demonstration: The setup used for experimental demonstration is a Rb atomic clock prototype based on the POP scheme as described in [2]. The durations of the three interrogation phases are set as follows: 1) optical pumping time T_p=0.4 ms; 2) microwave pulse time T_1=0.4 ms; 3) Ramsey time T_{Ramsey}=3 ms; 4) optical detection time T_d= 0.7 ms. The total cycle period is T_c =4.94 ms including some short pauses. Based on the duration settings, the POP clock's short-term stability is limited by the optical detection noise rather than that arising from the Dick effect [2]. In order to investigate the specific impact of the optical detection time τ_d at fixed T_C, we keep the detection light pulse duration T_d= 0.7 ms fixed in each experiment so that the instability contribution from the (microwave) Dick effect remains constant. But we vary the detection time τ_d from 0.1ms to 0.7ms. Such an optically real-time detected signal has a fast time constant of ~40 µs so that it is accurately approximated by *g(t)* as given in (1).

The central fringe contrast as function of τ_d and the optical detection signal's RIN have been measured and are shown in Fig. 2. In Fig. 2a, the contrast reduction with an increased detection time is due to the re-pumping effect during the detection phase. However, we still can obtain C≈35% at τ_d=0.7 ms. For all the measurements, the atomic quality factor is always Q_a=4.2×10^7 because of the fixed Ramsey time. Figure 2b shows the typical RIN performance for the detection light measured in DC mode, with the same detection light level as during detection phase of the POP clock operation.



a *b*

Fig. 2 *Measured contrasts as a function of detection time and the typical RIN performance.*
a Contrast fitting result: C = 37.4% – 3.97%/ms × τ_d
b RIN fitting result: RIN= $8.6\times10^{-13}\,\mathrm{Hz}^{-1}+1.3\times10^{-9}/f+5.4\times10^{-8}\,\mathrm{Hz}/f^2$.

Figure 3 depicts the theoretical and experimental stabilities at τ=1s for the POP Rb clock prototype, where the theoretical data is calculated from (8) and above measured parameters, with no free parameters. For comparison, two other results predicted for our clock by using the previous formulas in [1, 3] are also presented. All three theoretical curves include a (microwave) Dick-effect limit of $7.5\times10^{-14}\,\tau^{-1/2}$ which is independent of τ_d (see above). The theoretical values based on (8) are highly consistent with the measured ones and precisely reflect the stability behaviour with variations of the detection time with respect to the other two methods. This demonstrates that, to a certain degree, the impact of detection noise can be reduced by increasing the detection time which provides us another degree of freedom to optimize the clock's stability.

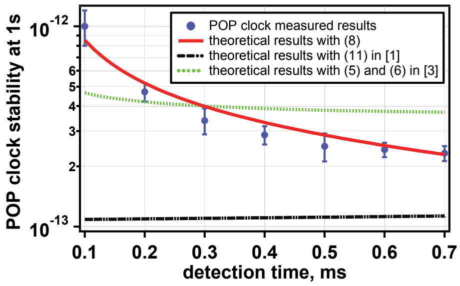


Fig. 3 *Theoretical and measured clock short-term stabilities at τ=1s as a function of detection time.*

Shot-noise Estimation: Often the shot noise – as a kind of quantum noise in atomic clocks –is submerged by technical noises. But the shot-noise limit as a physical quantity reflecting the atomic resonator's potential performance still deserves a precise evaluation. Since the RIN from optical shot noise can be defined as $S_{RIN}^{shot-noise}=2h\nu/(\eta P_0)$ (η is the quantum efficiency of the photodetector; ν and P_0 are the detection light's frequency and detected power, respectively), following (8), the optical detection shot-noise limit can be expressed as

$$\sigma_y^{shot-noise}(\tau)=\frac{1}{\pi}\frac{1}{C}\frac{1}{Q_a}\Big(\sum_{n=1}^{\infty}\mathrm{sinc}^2(\pi n f_c\tau_d)\frac{2h\nu}{\eta P_0}\Big)^{-1/2}\sqrt{\frac{T_c}{\tau_d}}\,\tau^{-1/2}\quad(9)$$

which is similar to the expressions given in [1, 10]. Table 1 gives the shot-noise limitations to short-term stabilities of the POP Rb clock prototype shown in Fig. 3, calculated with (9). As it can be seen, a reduction of the detection noise (RIN) would allow a short-term frequency stability well below $1\cdot10^{-13}\,\tau^{-1/2}$.

Table 1: Shot-noise stability limits for the POP clock as function of τ_d .

τ_d (ms)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$\sigma_y^{\text{shot-noise}}$ $(10^{-14}\,\tau^{-1/2})$	35.0	17.6	11.7	8.8	7.0	5.8	5.0

Conclusions: The stability limits of a Ramsey-type atomic clock arising from optical detection have been re-evaluated. In contrast to previous work, we derived an analytical expression of the stability that takes into account the duration of the detection pulse. It demonstrates the impact of detection time on the clock stability, offering us another method for clock stability optimization. The theory is in good agreement with the experimental results and is extended to also evaluate the optical shot-noise stability limit. The results are of relevance for the optimization and stability budgets of state-of-the-art Ramsey-type atomic clocks, including primary fountain clocks and CPT-based clocks.

Acknowledgment: This work was supported by the Swiss National Science Foundation (SNSF grant no. 140712) and the European Metrology Research Programme (EMRP project IND55-Mclocks). The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union. We also acknowledge previous support from the European Space Agency (ESA) and the Swiss Space Office (Swiss Confederation). We thank A. K. Skrivervik, C. Stefanucci (both EPFL-LEMA) and M. Pellaton and T. Bandi (both UniNe-LTF) for their contributions to realizing the cell and cavity, and C. Calosso (INRIM, Italy) for providing the microwave LO.

S. Kang, M. Gharavipour, C. Affolderbach and G. Mileti *(Laboratoire Temps-Fréquence (LTF), University of Neuchâtel, Avenue de Bellevaux 51, 2000 Neuchâtel, Switzerland)*

E-mail: gaetano.mileti@unine.ch

S. Kang: also with Key Laboratory of Atomic Frequency Standards, Chinese Academy of Sciences, Wuhan Institute of Physics and Mathematics, Wuhan 430071, China. S. K. acknowledges support from the National Natural Science Funds of China (Grant no. 11304360).

References

[1] Micalizio, S., Calosso, C. E., Godone, A., et al.: 'Metrological characterization of the pulsed Rb clock with optical detection', *Metrologia*, 2012, **49**, pp. 425-436.

[2] Kang, S., Gharavipour, M., Affolderbach, C., et al.: 'Demonstration of a high-performance pulsed optically pumped Rb clock based on a compact magnetron-type microwave cavity', *J. Appl. Phys*. 2015, **117**, 104510.

[3] Guerandel, S., Danet, J.-M., Yun, P., et al.: 'High performance compact atomic clock based on coherent population trapping', Proc. IEEE International Frequency Control Symposium, Taipei, Taiwan, 2014, pp. 1-5.

[4] Dick, G.J.: 'Local oscillator induced instabilities in trapped ion frequency standards', in Proc. 19th Precise Time and Time Interval Meeting, Redondo Beach, CA, 1987, pp.133-147

[5] Dick G.J., Prestage, J. D. , Greenhall, C. A., et al.: 'Local oscillator induced degradation of medium-term stability in passive atomic frequency standards', in Proc. 22nd Precise Time and Time Interval Meeting, Vienna, Virginia, 1990, pp. 487-508.

[6] Santarelli, G., Audoin, C., Makdissi, A., Laurent ,P., et al.: 'Frequency stability degradation of an oscillator slaved to a periodically interrogated atomic resonator', *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 1998, **45**, pp. 887-894.

[7] Audoin, C., Santarelli, G., Makdissi, A., Clairon, C., et al.: 'Properties of an oscillator slaved to a periodically interrogated atomic resonator,' *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 1999, **45**, pp. 877-886.

[8] Presti, L.L., Rovera, D., De Marchi, A.: 'A simple analysis of the Dick effect in terms of phase noise spectral densities', *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 1998, **45**, pp. 899-905.

[9] Riehle, F.: 'Frequency standards basics and applications', (Wiley-VCH, Weinheim, 2004).

[10] Liu, X., Mérolla, J.-M., Guérandel, S., et al.: 'Ramsey spectroscopy of high-contrast CPT resonances with push-pull optical pumping in Cs vapor', *Opt. Express*, 2013, **21**, (10), pp. 12451-12459.