Investment in an Uncertain Backstop: Optimal Strategy for an Open Economy

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Abstract

The present study examines the problem facing a resource-importing economy seeking to achieve energy independence by developing a renewable substitute. The invention of the substitute is assumed to follow a stochastic process which can be influenced by investment in energy R&D. I analyze the optimal investment strategy under alternative assumptions with respect to the economy's access to international financial markets, the terms on which credit is available, and the country's degree of dependence on resource imports. It is found that, in general, having access to capital markets does not necessarily lead to a higher investment rate. However, in the empirically-relevant range of the elasticity of intertemporal consumption substitution, the economy with access to credit invests more than under financial autarky. A higher degree of dependence on non-renewable resource imports implies a lower optimal investment. Arrival of the substitute does not necessarily lead to an immediate improvement in the net foreign asset position but may cause its further deterioration.

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1 Introduction

Private and public investment spending on projects devoted to research and development of renewable energy sources ("backstops") is often motivated by concerns about exhaustion of non-renewable energy resources, their ever increasing market price, pollution caused by the use of fossil fuels and climate change. If we look across countries at the leading investors in energy R&D in per capita terms, we find Japan occupying the first place (IEA 2006a). Not surprisingly, this country is also well known for its heavy dependence on energy imports.[1] Among European economies leading the way in terms of their share of national income devoted to renewable energy sources are Switzerland, Denmark, Finland, the Netherlands, and Sweden (IEA 2006b). These are again countries that do not possess large stocks of fossil fuels, making them heavily dependent on imports (except for the Netherlands which do possess large reserves of natural gas and Denmark, which is expected to continue its North Sea production of oil and gas in excess of its own demand until 2018.)[2]

The purpose of the present paper is to study the problem facing a resource-importing country, hereafter RIC, which seeks to achieve energy independence by developing a substitute for the non-renewable importable input. This is assumed to require sustained investment in an R&D program. Arrival of the substitute follows a stochastic process with the probability of a successful outcome per unit of time being a non-decreasing function of the rate of investment in R&D. Apart from trade in the resource market, RIC can also participate in the global financial market. This latter dimension is most often overlooked in the literature on backstop-technology adoption and resource management in general. As we shall see, however, access to international lending and borrowing is

[1] Although Japan is only the second largest oil importer after the United States, it meets a larger share of its energy needs through imports of oil than the U.S. does (U.S. Energy Information Administration, http://www.eia.doe.gov/country/index.cfm).

[2] Denmark is also a major producer and exporter of wind energy (see Sherman (2009)).

important in several dimensions, especially if a country is heavily dependent on imports of an essential factor of production.

The literature on backstop-technology adoption has its origins in the wake of the oil price shock of 1973. The early contributions focus on a closed economy, endowed with a known stock of an exhaustible resource, seeking to sustain its consumption in the long run by appropriately substituting a renewable backstop for the non-renewable essential input. The arrival date of the substitute is assumed to be either known with certainty or uncertain but governed by an exogenous stochastic process (see, e.g., Dasgupta and Heal 1974, Dasgupta and Stiglitz 1981). The seminal contribution of Kamien and Schwartz (1978) extends this analysis by endogenizing the uncertain arrival date through investment in R&D. Hung and Quyen (1993) go further to determine the optimal time to initiate the R&D project, although their R&D investment policy is simplified to a single-date expenditure, after which a backstop may arrive with a constant Poisson rate.[3] The work of Dasgupta, Gilbert and Stiglitz (1983) shows (in the context of a deterministic model) that the intention to develop a substitute and its eventual arrival can trigger a strategic response from resource owners. Harris and Vickers (1995) extend this analysis to a stochastic setting. More recently, Gerlagh and Liski (2011) and Jaakkola (2012) analyze dynamic interactions between resource importers and resource exporters.[4] Jaakkola (2012) and Ploeg (2012) add the climate change dimension to the analysis of substitute arrival and strategic behavior. Although these contributions are concerned with open economies, their analysis is limited to exchange of the resource for the consumption good, while the possibility of international lending and borrowing is ruled out.

[3] Tsur and Zemel (2003) propose an alternative (deterministic) framework of analysis, where the cost of the backstop falls continuously as the knowledge base accumulates through R&D. This ensures a continuous transition from the non-renewable to the backstop. Their model advocates an R&D policy characterized by the most rapid approach path to the target-knowledge process which should then be followed forever.

[4] See also Gerlagh and Liski (2012).

The trade-theoretic literature, on the other hand, deals with problems related to exhaustible resource management and, in some cases, for countries that have access to foreign credit, but it does not addressed the problem of optimal investment in the development of a backstop technology.[5] Moreover, these contributions consider purely deterministic models and therefore exclude the possibility of uncertainty affecting behavior.[6] The purpose of the present study is to bridge the existing gap between the closed-economy analysis of investment in a backstop and open-economy models of trade in goods and financial assets within a fully dynamic stochastic optimization framework. This will make it possible to examine the role of international financial markets in influencing optimal investment strategies in a stochastic environment, an issue of increasing importance in a world where energy prices and international indebtedness are becoming dominant themes.

In order to highlight the role of access to credit, I first present in Section 2 a model of a resource-importing economy which initiates development of an energy substitute under financial autarky. Section 3 extends the model to allow for international lending and borrowing. Section 4 solves the two models numerically and compares the optimal R&D investment rates, the time profile of consumption and the net foreign asset position before and after the invention of a substitute (if such happens to occur). Access to international lending and borrowing allows for a more efficient intertemporal allocation of resources and a higher lifetime welfare as compared with the case of financial autarky. While this is generally to be expected, a comparison of the optimal investment rates under financial autarky and access to foreign credit enables us to address a number of entirely new

[5]Kemp and Long (1984) do consider resource replacement but in a deterministic setting, where the resource price is exogenous and constant and there is no possibility to participate in the international financial markets. Djajić (1988) considers a two-country world, where both countries are endowed with some stock of the resource and can lend or borrow from each other at an endogenously determined rate of interest. The dynamics of his model are, however, limited to only two time periods and neither country intends to develop a backstop.

[6]An exception is Dasgupta, Eastwood and Heal (1978) who do consider uncertainty related to future energy demand. They also introduce a possibility to accumulate a foreign asset yielding a constant rate of return but focus on a resource-exporting economy, which is not engaged in any R&D activity.

issues. First, there is the question of how the degree of dependency on imported energy resources affects the economy's optimal investment in the development of a backstop. On the one hand, greater dependency makes it more urgent to discover a substitute. On the other hand, it also implies a larger import bill prior to invention, which tightens the economy's budget constraint and makes any given investment program relatively more burdensome. My analysis shows that for empirically plausible values of the elasticity of intertemporal consumption substitution, a higher price of non-renewables entails a lower investment rate in renewables R&D, while having access to credit is equivalent to a smaller share of non-renewables in final output production. The second set of issues concerns the role of the cost of credit which influences not only the time path of the country's net foreign asset position but the optimal investment decision as well. The paper concludes in Section 5 with a summary of the main results.

2 Financial Autarky

Let me introduce the assumptions and the notation by starting with the simplest case of financial autarky. Consider a resource-importing country (RIC) which produces a composite consumption good according to the production function

$$Y_t = F(R_t, L), \tag{1}$$

where $F(.,.)$ is a strictly increasing, concave and twice-differentiable function of both arguments, L is the constant labor input and R_t is the resource input, which must be entirely imported from abroad. The price of the resource, measured in terms of the consumption good, satisfies $P_t = P_0 e^{rt}$, P_0 known, and r is a constant growth rate. RIC wishes to develop a backstop, i.e., to invent and produce a substitute for the resource, but this requires setting up and maintaining an R&D lab.[7] RIC may invest $m_t \geqslant 0$ units of the consumption good each period to keep the lab operational. The discovery of a substitute follows a stochastic process which can be influenced by the investment decision. Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a probability space, and let τ be a random variable, which I call the arrival date of the substitute. I assume that the probability measure $\mathcal{P}$ depends on the investment rate in the following way

$$\mathcal{P}[\tau \in (t, t+dt) | \tau \geqslant t] = q(m_t)dt + o(dt),$$

where $q : \mathbb{R}^+ \to \mathbb{R}^+$ and its first derivative $q'(m_t)$ are continuous functions, $q(0) \geqslant 0$ and the limit of $o(dt)/dt$ is zero as $dt \to 0$.

If the backstop arrives, a known quantity B of the substitute becomes available every period at zero cost.[8] This quantity simply substitutes for the resource input in the production function. The flow of output is then constant and given by $\bar{Y} = F(B, L)$ and the resource is no longer imported.[9]

The social planner's objective is to maximize the expected lifetime welfare by choosing the optimal consumption rate, c_t, the investment rate, m_t, and imports of the resource,

[7] The model assumes that once the substitute is invented, RIC becomes its unique owner. This occurs, for instance, if the substitute (or its production process) is specific to RIC's geographic location or if RIC can patent the invention. I do not, however, analyze issues related to patent races.

[8] Allowing for a cost of production which is positive, constant or varying over time but exogenous, will merely affect the relevant budget constraint in a straightforward manner. The qualitative results will remain intact.

[9] If B is not large enough, however, it may be optimal to continue importing energy from abroad until its price rises sufficiently to reduce the demand to the available per-period supply of the substitute. In the present paper I do not analyze the optimal timing of the switch from the non-renewables to the backstop, which has already been studied elsewhere (see, e.g., Gerlagh and Liski (2011)) but wish to focus on the optimal investment strategy under uncertainty. In the rest of the analysis I therefore assume that B is sufficiently large, i.e., $B \geqslant g(P_0 e^{r\tau})$, $\forall \tau$, where the function $g(.)$ is the inverse of the marginal productivity of the resource. In particular, it is sufficient to assume that $\partial F(B, L)/\partial B \leqslant P_0$, so that it is no longer optimal to continue importing the exhaustible resource even if the substitute becomes available from the start. See Amigues et al. (1998) for treatment of a capacity constraint on the flow of the substitute.

R_t, given the constant rate of time preference, ρ, and the resource price path:

$$\max_{c_t, m_t, R_t} \quad \int_0^\infty \left\{ \int_0^\tau u(c_t) e^{-\rho t} dt + \int_\tau^\infty u(\tilde{c}) e^{-\rho t} dt \right\} f_\tau d\tau, \tag{2}$$

subject to the constraints

$$\begin{aligned} c_t &= F(R_t, L) - P_t R_t - m_t, \\ \tilde{c} &= \bar{Y}, \\ f_\tau &= q(m_\tau) e^{-\int_0^\tau q(m_s) ds}, \end{aligned} \tag{3}$$

where $u(.)$ is a strictly increasing, concave and twice-differentiable function with $\lim_{c \to 0} u'(c) = \infty$. The consumption rate in Phase II, i.e., after the discovery has taken place, is denoted by $\tilde{c}$. Note that once the substitute has arrived, there is no more need to maintain the R&D investment.

This stochastic control problem can be analyzed with the aid of the Hamiltonian (see Boukas et al. (1990) or the Appendix):

$$H = \left\{ u(F(R_t, L) - P_t R_t - m_t) + q(m_t) \frac{u(\bar{Y})}{\rho} \right\} e^{-\rho t - z_t} + \nu_t q(m_t), \tag{4}$$

where z_t is an auxiliary state variable such that $\dot{z}_t \equiv \frac{dz_t}{dt} = q(m_t)$, $z_0 = 0$, and ν_t is the associated co-state variable. The necessary conditions for optimality consist of

$$R_t : \quad u'(c_t) \left(\frac{\partial F_t}{\partial R_t} - P_t \right) e^{-\rho t - z_t} = 0, \tag{5}$$

$$m_t : \quad \left\{ -u'(c_t) + q'(m_t) \frac{u(\bar{Y})}{\rho} \right\} e^{-\rho t - z_t} + \nu_t q'(m_t) = 0, \tag{6}$$

$$z_t : \quad \left\{ u(c_t) + q(m_t) \frac{u(\bar{Y})}{\rho} \right\} e^{-\rho t - z_t} = \dot{\nu}_t \tag{7}$$

and the budget constraint (3). Eq. (5) is the efficiency in production condition, which requires that the marginal productivity of the resource input equals its price. Eq. (6) guarantees the optimality of investment by equating the present value of the marginal investment cost, $u'(c_t)e^{-\rho t - z_t}$, to the present value of the marginal expected benefit, $q'(m_t)\left[\frac{u(\bar{Y})}{\rho}e^{-\rho t - z_t} + \nu_t\right]$. Eq. (7) describes the dynamics of the co-state variable.

Given the structure of the production technology (1), eq. (5) relates the quantity of imports to the resource price as $R_t = g(P_t)$, where $g(.)$ is the inverse function of the marginal productivity of resource with $g'(.) < 0$. Define the net output as $Y_t^n \equiv F(R_t, L) - P_t R_t$. Then the value of Y_t^n at each point in time is determined by P_t:

$$Y_t^n = F(g(P_t), L) - g(P_t)P_t \tag{8}$$

with $\frac{\partial Y_t^n}{\partial P_t} = -g(P_t) < 0$, $\frac{\partial^2 Y_t^n}{\partial P_t^2} = -g'(P_t) > 0$. The budget constraint (3) may then be rewritten as

$$c_t = Y_t^n(P_t) - m_t. \tag{9}$$

Solving for ν_t from (6), differentiating with respect to time and inserting the result in (7) yields

$$\dot{m}_t = \frac{q'(m_t)}{q''(m_t)}\left[\frac{q'(m_t)[u(\bar{Y}) - u(c_t)]}{u'(c_t)} + \frac{u''(c_t)\dot{c}_t}{u'(c_t)} - \rho - q(m_t)\right],$$

which, in combination with (8) and (9), can be solved for the optimal time path of investment.

From this point on, let me assume for simplicity that the investment rate is time-invariant, i.e., $m_t = m$, $\forall t$, which means that RIC must commit itself to a certain constant expenditure per unit of time to keep the R&D lab operational.[10] Then the

[10] Hung and Quyen (1993) also use a fixed investment assumption, although in their setting R&D investment is modeled as a single expenditure at the initial point in time which determines the arrival rate of a substitute. By contrast, in the present analysis, m must be invested at each point in time, so that the sacrifice of current consumption becomes more and more difficult to support as the time goes by without the substitute being

optimal investment rate under financial autarky, m^{*FA}, solves

$$-\frac{u''(c_t)c_t}{u'(c_t)}\hat{c}_t = q'(m)\left[\frac{u(\bar{Y}) - u(c_t)}{u'(c_t)}\right] - \rho - q(m), \tag{10}$$

where the first term on the right-hand side corresponds to the economy's implicit rate of interest.

Total differentiation of eq. (10) yields the effects of change in the optimal investment rate due to a change in the key exogenous variables of the model:

$$\Delta_m dm = \Delta_P dP_0 + \Delta_r dr + \Delta_B dB + \Delta_\rho d\rho,$$

where

$$\begin{aligned}
&\Delta_m \equiv g(P_t)\dot{P}_t\left[\frac{u'''(c)u'(c) - (u''(c))^2}{(u'(c))^2}\right] - \frac{u(\bar{Y}) - u(c)}{u'(c)}\left[q''(m) + \frac{q'(m)u''(c)}{u'(c)}\right],\\
&\Delta_P \equiv \Omega\frac{dP_t}{dP_0}, \quad \text{where} \quad \frac{dP_t}{dP_0} = e^{rt} > 0, \quad \Omega = -r^2 e^{rt}\frac{u''(c)}{u'(c)}\left[g'(P_t)P_t + g(P_t)\right] +\\
&+\left\{q'(m)\frac{\left[(u'(c))^2 + (u(\bar{Y}) - u(c))u''(c)\right]}{(u'(c))^2} - g(P_t)\dot{P}_t\left[\frac{u'''(c)u'(c) - (u''(c))^2}{(u'(c))^2}\right]\right\} g(P_t),\\
&\Delta_r \equiv \Omega\frac{dP_t}{dr}, \quad \text{where} \quad \frac{dP_t}{dr} = tP_t \geqslant 0\\
&\Delta_B \equiv q'(m)\frac{u'(\bar{Y})\frac{\partial\bar{Y}}{\partial B}}{u'(c)} > 0,\\
&\Delta_\rho = -1,
\end{aligned}$$

The term Δ_m is, in general, of ambiguous sign. However, for standard utility functions employed in the literature, such as CRRA and negative exponential, the term $u'''(c)u'(c) - (u''(c))^2$ is non-negative,[11] while for a concave $q(m)$ function the term

invented.

[11] This term is equal to zero for the class of negative exponential functions of the type $u(c) = -e^{-\theta c}$ and for linear utility functions. It is strictly positive for negative exponential utility of the type $u(c) = -e^{1/c}$ and for widely used in the literature CRRA utility.

$q''(m)$ is negative. This is sufficient to ensure that $\Delta_m > 0$. The terms Δ_P and Δ_r are of ambiguous sign since $\Omega \gtrless 0$ and therefore the effects of P_0 and r on the optimal investment rate, i.e., $\frac{dm}{dP_0} = \frac{\Delta_P}{\Delta_m}$ and $\frac{dm}{dr} = \frac{\Delta_r}{\Delta_m}$ are ambiguous. This is hardly surprising. An increase in the resource price generates two conflicting effects: On the one hand, it tightens the economy's budget constraint as the import bill expands. On the other hand, it makes the development of the backstop more urgent as the economy's dependency on energy resources, whose market price rises exponentially, is increased. If the social planner of this economy is risk-neutral, we obtain

$$\frac{dm}{dP_0} = -\frac{q'(m)g(P_t)e^{rt}}{q''(m)\frac{u(\bar{Y})-u(c)}{u'(c)}} > 0, \quad \frac{dm}{dr} = -\frac{q'(m)g(P_t)tP_t}{q''(m)\frac{u(\bar{Y})-u(c)}{u'(c)}} \gtrless 0,$$

where the numerators are unambiguously non-negative and the denominators are negative if $q(m)$ is concave or m lies in the concave region of $q(.)$. A risk-neutral planner will therefore react to an increase in the resource price or its growth rate by increasing investment in R&D. The effect of a change in the rate of time preference is given by $\frac{dm}{d\rho} = \frac{\Delta_\rho}{\Delta_m} < 0$, so that patient economies will tend to choose a higher investment rate. An increase in the flow of the backstop unambiguously calls for an increase in the R&D investment rate: $\frac{dm}{dB} = \frac{\Delta_B}{\Delta_m} > 0$. We will be able to gain more insight about how the optimal investment rate responds to variations in B, P_0, ρ, r, and other variables, such as the elasticity of intertemporal consumption substitution, in Section 4, where the model is solved numerically.

Transactions with the rest of the world have been limited so far to the exchange of the consumption good for the resource. I examine next how the optimal investment strategy is affected if RIC has the possibility to lend and borrow in the international financial markets. It is clear that access to a riskless saving technology allows to implement a smoother optimal consumption path. However, the following questions remain: To what extent does access to foreign credit alleviate the burden of investment, facilitating development of a more ambitious project? What role does foreign credit play when RIC's dependency on energy imports is increased? What is the role of the cost of credit? What is the optimal time profile of the net foreign asset position and how is it affected by the arrival of the backstop? Sections 3 and 4 address these and other related questions.

3 Access to World Financial Markets

In this section I allow RIC to have access to international financial markets, where a single riskless asset, denominated in units of the consumption good, is costlessly traded. The asset yields a constant world rate of return, r.[12] By arbitrage, the growth rate of the resource price must also be equal to r, assuming that extraction is costless (Hotelling, 1931).

Let a_t denote RIC's net foreign asset position at time t. Assuming that the time horizon is infinite, the budget constraints in the first and the second phases, respectively, are

$$\dot{a}_t = F(R_t, L) - c_t - P_t R_t - m + r a_t, \quad \forall t \in [0, \tau), \quad a_0 \quad \text{given}, \tag{11}$$

$$\dot{a}_t = F(B, L) - \tilde{c}_t + r a_t, \quad \forall t \geq \tau, \tag{12}$$

$$\lim_{t \to \infty} a_t e^{-rt} = 0. \tag{13}$$

Eq. (11) states that during the first phase, while the substitute is not yet available, the rate of accumulation of foreign assets is equal to the total output minus expenditure on consumption, resource imports and investment, plus interest earned (paid) on the accumulated assets (outstanding debts). Eq. (12) states that during the second phase,

[12] Treating r as exogenous is based on the assumption that RIC's borrowing to finance (in part) its R&D efforts does not have a perceptible impact on the world rate of interest. Given the size of the global financial markets in relation to that of a major investment project in any one country, this assumption is arguably the most appropriate.

the change in the asset position is just equal to the constant flow of output minus consumption plus interest, and the resource is no longer imported. RIC's objective is to maximize (2) subject to (11) - (13).

The solution method consists of two steps. First, the maximized value of discounted (time-τ) welfare in Phase II is obtained, given the net foreign asset position at $t = \tau$. I call this function $\Phi(a_\tau)$. Then, the total lifetime welfare is maximized, given the relationship between a_τ and the welfare in Phase II (detailed derivation is relegated to the Appendix).

Consider the optimization problem pertaining to Phase II and assume a standard CRRA utility $u(x) = \frac{x^{1-\theta}}{1-\theta}$, where $1/\theta$ is the elasticity of intertemporal consumption substitution (EICS). RIC seeks to maximize

$$\int_{\tau}^{\infty} u(\tilde{c}_t)e^{-\rho(t-\tau)}dt \tag{14}$$

subject to (12) - (13) and a_τ given. The solution for the optimal $\tilde{c}_t$ is obtained in a straightforward manner using the standard dynamic optimization technique:

$$\tilde{c}_t = \tilde{c}_\tau e^{\frac{r-\rho}{\theta}(t-\tau)}, \quad \forall t \geq \tau, \quad \tilde{c}_\tau = \left(r - \frac{r-\rho}{\theta}\right)\left(a_\tau + \frac{\bar{Y}}{r}\right). \tag{15}$$

Then, the maximized value of (14), is

$$\Phi(a_\tau) = u(\tilde{c}_\tau)\Big(r - \frac{r-\rho}{\theta}\Big)^{-1}.$$

The Hamiltonian, associated with RIC's original optimization problem may then be written as

$$H = \Big\{u(c_t) + q(m)\Phi(a_t)\Big\}e^{-\rho t - z_t} + \eta_t\big[ra_t + F(R_t, L) - c_t - P_t R_t - m\big] + \nu_t q(m),$$

where η_t is the co-state variable associated with the constraint (11) and z_t is the auxiliary state variable, as in Section 2. The solution is implicitly given by the system:

$$-\frac{u''(c_t)c_t}{u'(c_t)}\hat{c}_t = r - \rho - q(m)\left[1 - \frac{u'(\tilde{c}_t)}{u'(c_t)}\right] \tag{16}$$

$$\tilde{c}_t = \left(r - \frac{r-\rho}{\theta}\right)\left(a_t + \frac{\bar{Y}}{r}\right), \tag{17}$$

$$q'(m)\left[\rho\Phi(a_t) - u(c_t) - u'(\tilde{c}_t)\dot{a}_t\right] = u'(c_t)r + q(m)u'(\tilde{c}_t), \tag{18}$$

$$\dot{a}_t = F(R_t, L) - c_t - P_t R_t - m + r a_t, \quad a_0 \text{ given.} \tag{19}$$

Eq. (16) describes the growth rate of consumption in Phase I. Note that if there is no uncertainty, the last term vanishes and the standard Keynes-Ramsey rule applies. When $q(m) > 0$, the standard rule is modified to account for the effect of uncertainty. The term in the square brackets is unambiguously positive since $\tilde{c}_t > c_t$ and therefore consumption grows at a lower rate, as compared to the certainty case. The lower optimal growth rate (or a more rapid decline) of consumption results in a higher dissaving rate at the beginning of the planning horizon in anticipation of the possible technological break-through. Moreover, the higher the flow of the substitute, B, in the event of a discovery, the lower the consumption growth rate and the higher the dissaving rate at the beginning of the planning horizon.

Eq. (17) determines the time-τ consumption rate, i.e., the consumption rate to which the economy jumps at the moment when the backstop arrives. It depends negatively (positively) on the stock of debt (assets) accumulated up to the time of the invention.[13] From time τ onwards the consumption rate during Phase II is no longer constant, as it was under financial autarky, but grows or contracts depending on the difference between the world rate of interest and RIC's rate of time preference, satisfying the standard Keynes-Ramsey rule. Without access to credit, Phases I and II were disconnected, in

[13] Convergence of the integral in (14) requires that $\frac{r-\rho}{\theta} - r < 0$, so that $\partial\tilde{c}_\tau/\partial a_t = r - \frac{r-\rho}{\theta} > 0$.

the sense that the optimal consumption rate in Phase II was independent of the variables pertaining to Phase I.[14] In the present setting, the two phases are connected through the net foreign asset position held at the time of invention. Eq. (18) is the optimality condition for the choice of m, which states that the marginal expected benefit from undertaking the investment must be equal to the marginal cost, which also includes the opportunity cost of not investing in the capital markets. The system (16) - (19) is solved numerically and analyzed in the next section.

4 Numerical Illustration and Discussion

This Section compares the solution to RIC's problem with access to credit (AC, for short) with the one under financial autarky (FA, for short). The objective is to analyze how the economy's dependence on energy resources translates into the choice of m and what role access to international capital markets plays in this respect. I also examine the optimal borrowing/lending strategy in an uncertain environment.

Let the production function be of Cobb-Douglas type: $Y_t = AR_t^{\alpha}L^{1-\alpha}, \quad 0 < \alpha < 1, \quad A > 0$. I assume that the invention of the substitute follows a Poisson process with the arrival rate $\lambda(m)$. The arrival rate is positively related to the R&D investment rate, i.e. $\lambda'(m) > 0$. It is assumed that $\lambda''(m) > 0$ for $m < \bar{m}$ and $\lambda''(m) < 0$ for $m > \bar{m}$. That is, when the investment rate is relatively small, commitment to an additional unit of sustained investment has an increasing marginal impact on the probability of making a discovery. Alternatively, when the investment rate is already high, the impact of an extra unit on the arrival rate is diminishing.[15] A natural candidate for the λ-function is a

[14]This is the reason why Kamien and Schwartz (1978) are able to summarize the value function pertaining to Phase II by the variable W which is taken to be exogenous and, more importantly, independent of the arrival date of the backstop.

[15]In the model of Kamien and Schwartz (1978) it is assumed that the probability of discovering a substitute

sigmoid-type function since it possesses the property that I have just outlined: convexity up to a certain (inflection) point and concavity thereafter. I specify the exact functional form for $\lambda(m)$ to be

$$\lambda(m) = \left(T_{min} + e^{(\mu-\gamma m)/\sigma}\right)^{-1}, \tag{20}$$

where $T_{min} \geqslant 0$ is the shortest possible time needed for the development of a backstop, and μ, γ, and σ are positive parameters calibrated as $\mu = ln(T - T_{min})$, $\gamma = 15$, $\sigma = 1$. A higher (lower) γ makes the slope steeper (flatter). The chosen values of μ and σ ensure that $\lambda(0) = 1/T$, where T is the length of the economy's planning horizon. This latter condition states that if the economy chooses a zero investment rate, there is still a chance of discovering a backstop once in T units of time. The inflection point $\bar{m} = \frac{\mu - \sigma \ln T_{min}}{\gamma}$.

The parameter values for the benchmark simulation are presented in Table 1. Labor

Labor	L	1
Technological parameter	A	1
Resource share	α	0.1
Substitute flow	B	0.5
Elasticity of marginal utility	θ	0.75
Rate of time preference	ρ	0.02
Resource price growth rate	r	0.02
Initial resource price	P_0	1
Initial asset holdings	a_0	0
Planning horizon	T	200
Minimum time to discover	T_{min}	20

Table 1: Benchmark parameter values.

input, the level of technology, and the initial resource price are normalized to unity. The

depends on the *cumulative* R&D effort. The rate of growth of R&D effort is, in turn, a concave function of investment. In their suggestions for possible extensions K&S write that "successful development of a new technology may require a sustained commitment of resources above a minimal level." Here I follow this route in assuming that the probability of inventing a substitute depends on the *level* of the sustained investment rate as opposed to cumulative investment.

share of exhaustible resources in the production function is assumed to be 10%.[16] The value of θ is calibrated so as to guarantee that the value of the elasticity of intertemporal consumption substitution lies in the empirically relevant range (see Epstein and Zin 1991, and Hansen and Singleton 1982, Keane and Wolpin 2001, Vissing-Jørgensen 2002). Multiple calibrations of θ are examined, especially in the analysis of the relationship between energy dependence and investment choice. The rate of growth of the resource price in the world market, as well as the rate of time preference, ρ, are set at 2% per annum.[17] The length of the planning horizon, T, is assumed to be 200 years,[18] while the minimum average time needed to discover a substitute is 20 years. The value of B is calibrated in such a way that it no longer pays to import the resource when B becomes available: $\partial Y/\partial B = A\alpha B^{\alpha-1}L^{1-\alpha} \leqslant P_0$.

4.1 Solution for the Optimal R&D Investment

The optimal investment rate is such that it maximizes expected lifetime welfare, given the planning horizon. Figure 1 plots RIC's expected welfare as a function of investment under financial autarky (thin line) and with access to credit (thick line). The maximum under AC occurs at $m^{*AC} = 0.2646$ and under FA at $m^{*FA} = 0.2477$. The optimal investment rate, as well as the associated expected welfare level, are higher and the average time to discover the backstop is lower under the second scenario (AC) as compared to the first scenario (FA). The result that $m^{*AC} > m^{*FA}$ is not general, however, and depends

[16] A relatively high value of the resource share is chosen in order to highlight the economy's dependency on energy input. Simulation results for alternative values of α are discussed in Section 4.2.

[17] We ignore the possibility that RIC's investment project might alter the time path of the resource price on the global markets. Even the recent nuclear incident in Japan did not seem to have an impact on the price of non-renewable energy resources in spite of it having triggered a large drop in planned investment in nuclear power plants across a number of major economies, including Germany, Switzerland and Japan.

[18] Although the model is written as an infinite horizon problem, the simulations are performed for a finite horizon. The numerical algorithm is based on (2). However, given the finiteness of the horizon, the truncated PDF is used: $f_\tau = \frac{\lambda(m)e^{-\lambda(m)\tau}}{1-e^{-\lambda(m)T}}$.

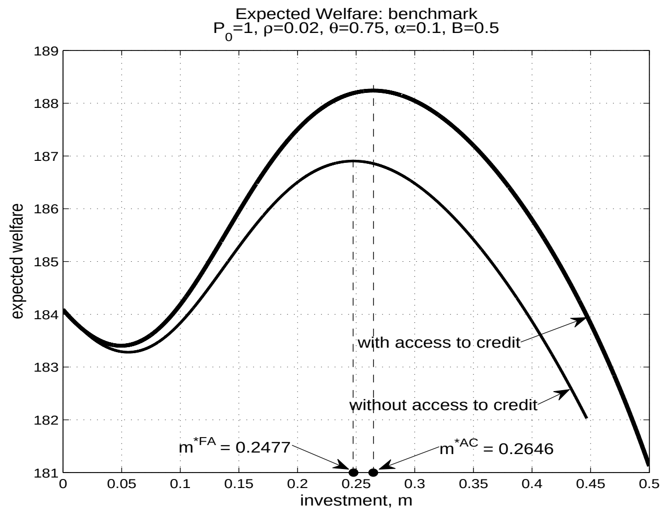


Figure 1: Expected welfare.

crucially on the elasticity of intertemporal consumption substitution, as will be discussed in the next subsection.[19]

The following subsections provide a more detailed analysis of the optimal investment choice under the two scenarios and, in particular, of how it is affected by a change in the economy's dependence on non-renewables or cost of credit. I also discuss the optimal path of the net foreign asset position.

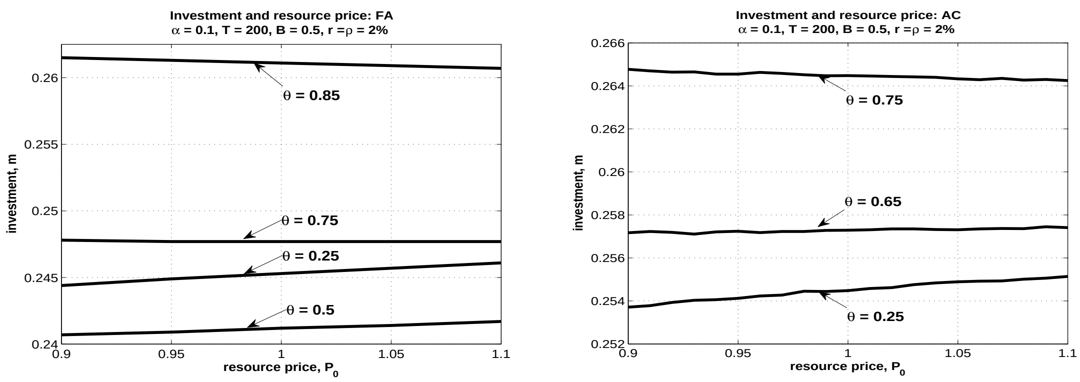
4.2 R&D Investment and Energy Dependence

Two interesting questions emerge at this point: First, does greater dependence on resource imports raise or lower the optimal investment rate, and second, what is the role of access to foreign credit in this respect? On the one hand, greater dependence makes it more urgent to develop an alternative source of energy. On the other hand, a country

[19]Note that due to the chosen specification of the function $\lambda(m)$, commitment to a relatively small amount of R&D expenditure ($m < 0.09$ under AC and $m < 0.11$ under FA) leads to a lower expected lifetime welfare than with no investment at all (see the drop in welfare for relatively low values of investment expenditure).

that is more dependent spends a larger share of its GDP on resource imports. It's budget constraint is then tighter, making the burden of any investment project relatively heavier. In terms of the present model, either a higher growth rate of the resource price, r, or a larger initial price, P_0, or a larger distributive share of resources in the production function, α, manifest themselves in a greater dependence on resource imports. For the moment I shall consider only the two latter parameters and discuss the role of r in subsection 4.4.

The optimal response of the investment rate to an increase in energy dependence hinges to a large extent on the planner's willingness to forgo current consumption in exchange for the prospect of having a higher consumption in the future, i.e., on the elasticity of intertemporal consumption substitution (EICS for short). It has already been established analytically in Section 2 that with an infinite EICS, the optimal m increases when either P_0 or r rise. The response of m is different, however, when EICS is reduced to the empirically relevant range. It matters as well whether the country has access to international capital markets or not. Figures 2a and 2b plot m^* as a function of P_0 for several values of θ (the inverse of EICS) under "financial autarky" and "access to credit", respectively. The slope of the relationship between m^* and P_0 changes from



(a) Energy dependence under FA

(b) Energy dependence under AC

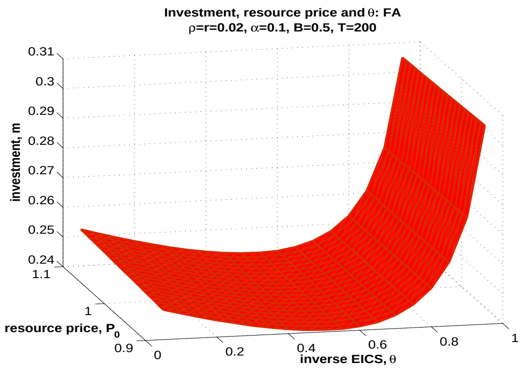
Figure 2: Price of non-renewables and optimal investment.

positive to negative as θ increases (i.e., EICS falls). To understand the intuition here, it is useful to think of the investment program as a plan to purchase, at every point in time, options that offer the prospect of stabilizing future consumption at a particular high level starting from an uncertain future date. Buying a larger number of options at each instant in the first phase advances the expected date of the payoff and lowers the chances of experiencing very low consumption rates at the end of Phase I. With a lower θ (i.e., higher EICS), the willingness to purchase such options diminishes as consumers care relatively less about the time profile of consumption as opposed to the total discounted consumption over the entire planning horizon.

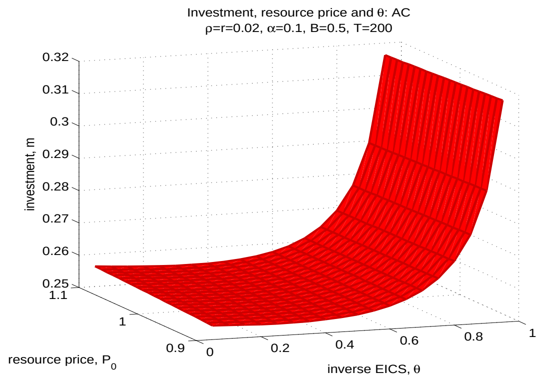
An increase in P_0 lowers the economy's expected net income over the planning horizon. This raises the utility cost of investing at any given rate in the development of the backstop. At the same time, with a higher P_0, the benefit of investing is also greater, in the sense that the expected future jump in income that the investment program helps bring forward in time, is larger. The impact of an increase in P_0 on the cost relative to the benefit of marginal investment is larger in the case of a higher degree of concavity of the utility function. Thus, when θ is relatively higher, the optimal investment rate declines with P_0, while with a lower θ, investment is positively related to P_0. Empirical studies of EICS conclude that the relevant range is below 2 which corresponds to $\theta > 0.5$.[20] Our numerical results show that in the benchmark calibration m^* is decreasing in P_0 for $\theta > 0.75$ under FA and for $\theta > 0.65$ under AC. Therefore, the optimal response of the R&D investment rate is more likely to be negative as the non-renewable resource price rises.

In order to obtain a more comprehensive picture of the relationship among m^*, P_0, and θ, I plot in figures 3a (FA case) and 3b (AC case) a 3-dimensional surface with m^* on

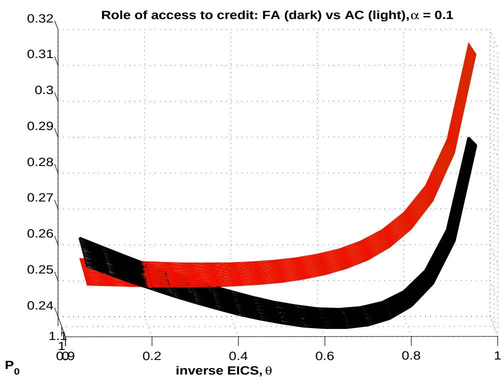
[20] Vissing-Jørgensen (2002) estimates EICS for stock- and bondholders, distinguishing among 3 wealth groups, as well as for non-stockholders. Her estimates range from 0.29 for stockholders to 1.38 for bondholders with higher estimates for top wealth layer households and close to zero estimates for non-stockholders. See also Epstein and Zin (1991) and Hansen and Singleton (1982).



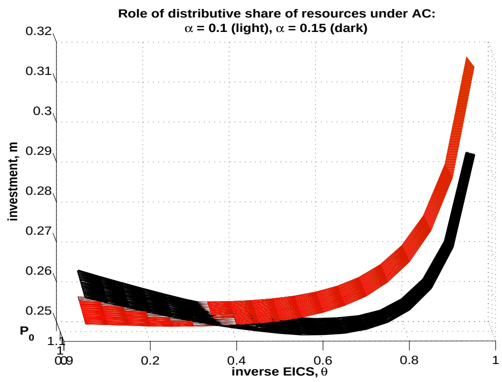
(a) Role of EICS under FA.



(b) Role of EICS under AC.



(c) Role of access to capital markets.



(d) Role of resource share in production, α.

Figure 3: Energy dependence under FA and AC.

the vertical axes and P_0 and θ on the two horizontal axes. Figure 3c superimposes both surfaces in one graph and demonstrates that the economy which has access to foreign capital markets does not necessarily invest more in renewables R&D as compared to an economy under FA: for high EICS (low θ) m^{*FA} (dark surface) is above m^{*AC} (light surface). In the empirically-relevant range of EICS, however, $m^{*AC} > m^{*FA}$, so that having access to credit does help sustain a higher investment rate.

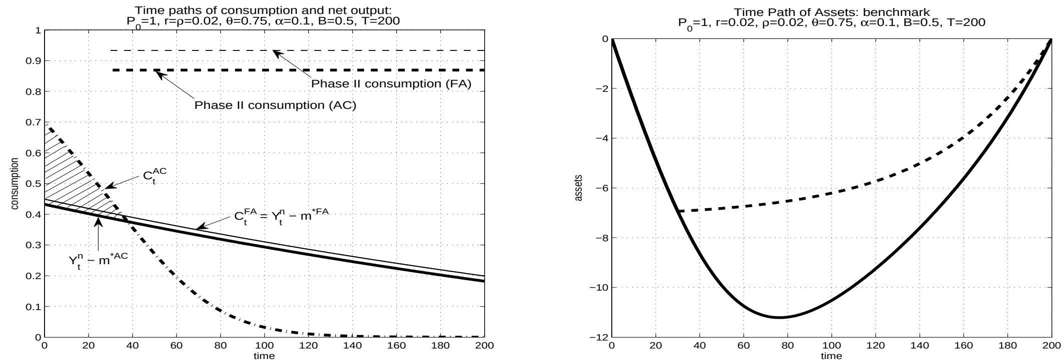
Finally, an increase in energy dependence may also be interpreted as an increase in the resource-use share in output production, i.e., a higher α. The optimal response of m to an increase in α (for a given P_0) also depends on θ, as shown in figure 3d. The light surface is the same as in figure 3b while the dark surface corresponds to α raised from 10 to 15%. Comparison of figure 3c with 3d reveals that having access to credit is equivalent to having a lower distributive share of energy resources in production of final goods.

To summarize the results so far, (a) when the non-renewable-resource price rises, the optimal response of renewables R&D investment is to fall due to the contractionary effect on the budget constraint; (b) the optimal investment rate in an economy with access to capital markets is higher than in an economy without such access, provided the elasticity of intertemporal substitution is not too high; (c) having access to credit is equivalent to being less dependent on non-renewable energy resources for production of final goods.

4.3 Optimal Paths of Consumption and Assets

The possibility of international lending and borrowing has important implications for the intertemporal allocation of consumption in an economy striving to achieve energy independence. Under the benchmark set of parameters, the economy is a net debtor. Borrowing from abroad (net of interest payments) can be visualized in figure 4a by the gap between the "c_t^{AC}" locus and the "$Y_t^n - m^{*AC}$" locus (the shaded area). The figure demonstrates that foreign credit has a dual purpose. It serves not only to finance the increase in the optimal investment rate but also to raise current consumption during the initial phase of the economy's planning horizon. This initial "over-consumption" is exactly the opposite of the precautionary saving phenomenon when an economy is subject to a random *drop* on income.

Note that in the present calibration the economy's rate of time preference, ρ, is identical to the rate of interest, r. In a deterministic environment, the economy's time path of consumption would have been flat. In a stochastic environment, however, the prospect of an upward jump in income results in a clockwise rotation of the consumption



(a) consumption (b) assets

Figure 4: Optimal paths of consumption and net foreign asset position.

path. During the initial phase of the planning horizon c_t^{*AC} exceeds $Y_t^n - m^{*AC}$, so that $\dot{a}_t - ra_t < 0$. Thus, in spite of ρ being equal to r, RIC's asset position initially deteriorates. This is shown in figure 4b.

Suppose that the invention happens to occur at $t = 30$.[21] Under both scenarios, consumption in figure 4a jumps upwards (see dashed lines) and the economy switches from borrowing to repaying its debt (the dashed line in figure 4b). A higher value of B obviously causes a larger upward jump and a faster loan repayment (not shown in the figures). Note that the initial consumption rate in Phase II is *higher* under FA than it is with AC. The reason is that under both scenarios the arrival of the backstop ensures a constant flow of output but under the second scenario the economy starts Phase II with a negative foreign asset position, which must be liquidated by the end of the planning horizon. In general, the initial consumption rate in Phase II and the subsequent time path of consumption under AC depend on the parameters of the model and in particular on the difference between r and ρ (see eq. (17)). If $r > \rho$ ($r < \rho$), consumption in Phase II exhibits a rising (falling) time path, while $\tilde{c}_\tau$ is below (above) the value obtained with $r = \rho$.

[21] With $m^{*AC} = 0.2646$, the probability of the discovery occurring by $t = 30$ is equal to 72.25%.

4.4 Role of the Cost of Credit

4.4.1 Evolution of net foreign asset position

So far the analysis proceeded under the simplifying assumption $\rho = r$, i.e., the economy's rate of time preference equals the world rate of interest. Variations in the cost of borrowing clearly affect RIC's optimal R&D investment rate, as well as its borrowing/lending decision. Interestingly, under specific conditions discussed below, RIC may find it optimal to have a positive net asset position (to be a lender) and at the same time maintain a relatively high R&D investment rate (above the rate under financial autarky).

Let us examine the role of the world interest rate in more detail. Figure 5 shows the time path of asset holdings under two alternative calibrations: a) the thin lines correspond to $r = 2.5\%$ (50 basis points above the benchmark) and b) the thick lines are drawn for $r = 3\%$ per year. The solid lines illustrate the evolution of assets under the assumption that the substitute is never discovered, while the dashed schedules are drawn assuming that the discovery takes place at $t = 60$ for case (a) and at $t = 100$ for case (b). Note that in spite of the fact that $r > \rho$, the economy is initially a net borrower under calibration (a). This is due to the effect of uncertainty, which, as we have seen in eq. (16), tilts clockwise the time path of consumption in Phase I and thus contributes to dissaving. Only if the substitute is eventually invented, may RIC become a lender (see the shaded area), with the length of the lending span depending negatively on the invention date and positively on EICS and on the difference between r and ρ. The later the substitute arrives, the longer the period of borrowing and the shorter the subsequent period of lending (if it exists at all). Note, in addition, that the larger is r relative to ρ, the weaker is the incentive to borrow during Phase I. Thus for higher world interest rates, the borrowing phase becomes shorter or even disappears, while the lending phase widens. Interestingly, for high enough r the borrowing phase may not necessarily occur at the beginning of the planning horizon. As illustrated by the thick

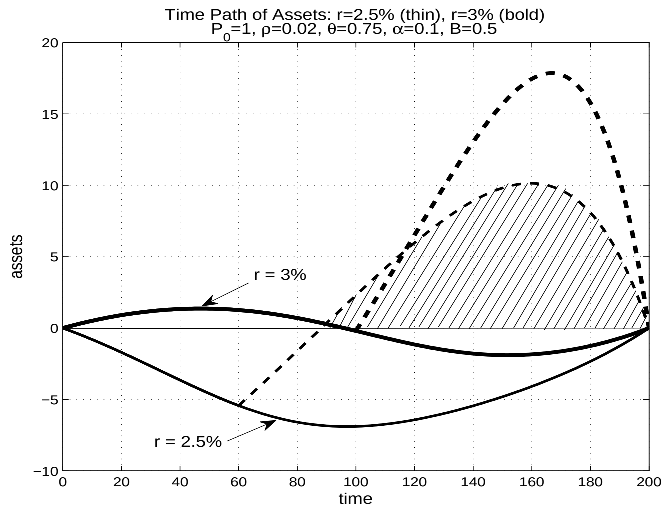


Figure 5: Asset position and the intertemporal terms of trade:
$r = 0.025$ (thin lines), $r = 0.03$ (thick lines).

solid line, for $r = 3\%$ RIC is initially a net lender in spite of maintaining a relatively high investment rate. The net asset position in this case exhibits a wave-shaped time profile with borrowing phase occurring at the end of the planning horizon. If r is relatively high and the invention occurs relatively late, the time profile of the net asset position peaks twice, as in the case of calibration (b) where the invention occurs at $\tau = 100$.

4.4.2 Invention date and debt repayment

So far we have seen that the arrival of the substitute initiates repayment of the debt or further improves the asset position if it is positive: immediately after the invention the dashed lines are positively sloped and lie above the solid schedules (see figures 4b and 5). This, however, may not always be true. The optimal time path of the net foreign asset position after the invention depends on the relationship between r and ρ. It is clear that when $r < \rho$, the economy will consume at a declining rate during Phase II, i.e., $\hat{\tilde{c}}_t = \frac{r-\rho}{\theta} < 0$. Moreover, the difference between the market rate of interest and RIC's rate of time preference also affects the *initial* consumption rate in Phase II: the smaller (i.e., more negative) is $r - \rho$, the larger is $\tilde{c}_{\mathcal{T}}$ (see eq. (17)). When r is sufficiently below ρ, the economy will in fact find it optimal to start Phase II with a consumption rate in excess of its income (net of interest and imports) which entails a further deterioration of the net asset position. This is illustrated in figure 6, where r is set at 0.5%. As

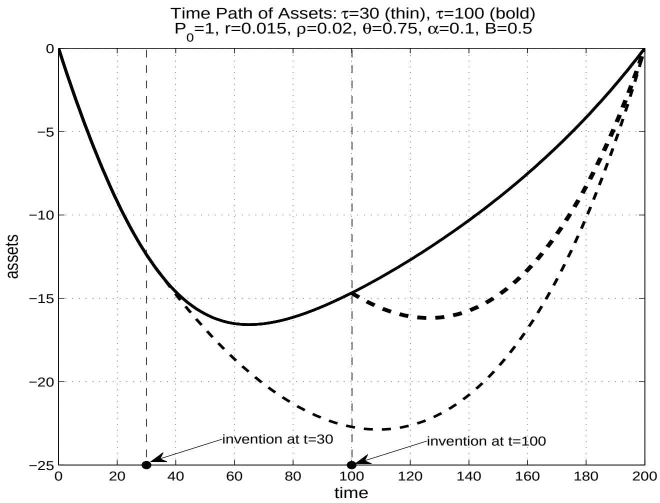
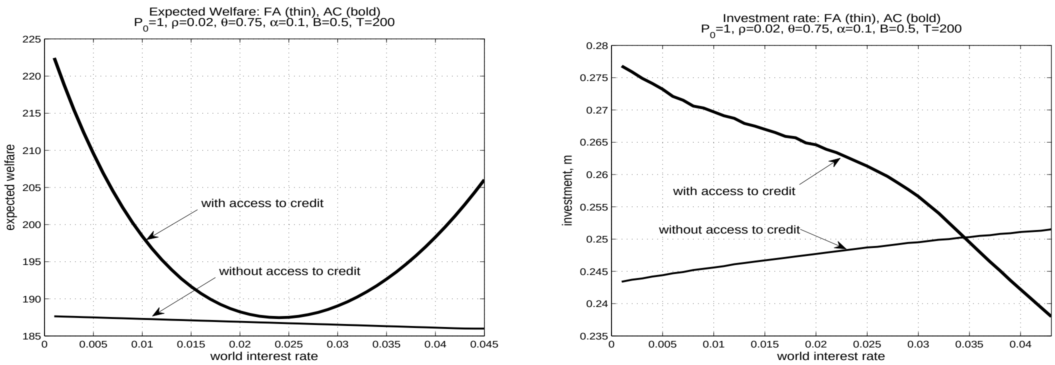


Figure 6: Deterioration of the asset position after the invention.

before, the solid line is drawn under the assumption that the substitute never arrives, while the dashed lines are drawn assuming that the invention occurs at $t = 30$ (thin dashed line) and $t = 100$ (thick dashed line). Given that credit is relatively cheap, the net asset position continues to deteriorate right after the invention and reaches the minimum several years later than if the substitute had never arrived. Throughout the remainder of the planning horizon the economy is more indebted than it would have been without the invention. Interestingly, the time profile of a_t may exhibit a double-trough pattern when invention occurs relatively late as, for example, at $\tau = 100$. The high levels of indebtedness, equal to a multiple of the economy's GDP, are nonetheless perfectly sustainable. This is true even if the backstop never arrives. In this case, the debt is repaid at the expense of current consumption which falls over time.

4.4.3 Cost of credit and lifetime welfare

Further examination of the role of the market rate of interest leads us to consider its effect on the economy's expected lifetime welfare. When access to credit is available, r affects expected welfare through two channels. The first one is the resource price: The higher the rate, the greater the rate of increase in P_t and the heavier the burden of future payments for resource imports. An additional channel emerges with the possibility of lending and borrowing. If RIC is a net borrower, a higher r implies a heavier debt burden, so that both effects contribute to a lower expected welfare. On the other hand, if RIC is a net lender, a higher r represents an improvement in its intertemporal terms of trade, contributing to higher expected welfare. Whether RIC is a borrower or a lender, is determined endogenously and depends on the structure of its preferences and its production technology, on the amount of the substitute it expects to obtain in the case of a technological breakthrough, and finally on the relationship between r and ρ. Thus, in general, the net effect of the world interest rate on the economy's expected welfare is ambiguous. It depends, in essence, on the volume of its trade in the resource market in relation to the volume of its net lending over the entire planning horizon. It is generally to be expected, however, that an economy's welfare is higher with free trade, in this case trade in the financial asset, than it is under autarky. This is illustrated in figure 7a, where I show RIC's expected lifetime welfare, under the optimal investment strategy, as a function of the market interest rate, holding other parameters at their benchmark levels. Under financial autarky the expected welfare declines with r, as shown by the thin line. In this case, only the effect of r on the price path of the non-renewable resource impinges on welfare. With access to credit the schedule is U-shaped, reflecting the conflicting forces discussed above. Note that regardless of the value of r,



(a) welfare

(b) investment

Figure 7: The effect of intertemporal terms of trade on welfare and investment rate.

the expected lifetime welfare with access to credit is always higher than without such access. The possibility of improving the efficiency of the intertemporal allocation of resources by transacting in the international financial market has therefore an important welfare-enhancing role.

The effect of the cost of credit on the optimal R&D investment rate can be visualized in figure 7b. First, observe that m^{*FA} is increasing in r, while m^{*AC} is decreasing in r. This difference in the optimal response hinges on the dual role of the interest rate in the latter scenario. Second, for high enough interest rate in relation to ρ, m^{*FA} exceeds m^{*AC}. In other words, economies which have access to capital markets but face a relatively high cost of credit tend to choose less ambitious investment projects as compared to what they would have chosen without access to credit. This is because holding foreign assets yielding a relatively high rate of return effectively substitutes for investment in R&D. The economy then chooses a lower investment rate and becomes a net lender in the capital market.

5 Conclusion

The paper attempts to answer three main questions: (i) What is the optimal investment rate in an R&D project which may secure a given flow of income in the future, with the probability of the success being dependent on the investment rate? (ii) How is the optimal investment choice affected by a higher degree of dependence on imports of an essential non-renewable factor of production? (iii) To what extent access to capital markets matters for the investment decision? The answers to these questions are analyzed in the context of a model of a resource-importing country (RIC), which seeks to achieve energy independence by developing a renewable substitute for a non-renewable essential input. I assume that the invention of a substitute follows a stochastic process which can be influenced by the appropriate investment in R&D. The focus of the paper is on the role of access to international lending and borrowing for the optimal choice of the economy's consumption and investment rates under uncertainty. This role is highlighted by comparing the outcomes under two extreme assumptions about the economy's access to global capital markets: financial autarky vs. full access.

With access to foreign credit the economy chooses a very different time path of consumption from the one obtained under financial autarky. Due to the presence of uncertainty, i.e., a possibility of a successful R&D outcome, the economy dissaves during an initial phase of its planning horizon and runs a negative foreign asset position, even when the rate of interest is above the rate of time preference. This type of behavior is exactly the opposite of precautionary saving in an environment with negative income shocks (see, e.g., Toche (2005) for the case of a job loss).

When it comes to the optimal choice of the R&D investment rate, having access to capital markets does not necessarily imply that the economy systematically invests more than it does without such access. The outcome depends crucially on the value of the elasticity of intertemporal consumption substitution (EICS). Numerical simulations show, however, that for empirically relevant range of EICS, R&D investment rate with access to credit markets always exceeds the investment rate under financial autarky.

Another key element influencing the optimal choice of the R&D investment rate is the economy's dependence on foreign energy sources, as measured by the share of GDP absorbed by the expenditure on non-renewable resource imports. In the context of the present model, energy dependence is determined by the market price of the resource and the distributive share of energy in the production of final goods. An increase in the resource price may either boost or decrease the investment rate depending on EICS. The numerical results show that in the empirically relevant range of values for EICS an increase in the resource price leads to a lower optimal investment rate. This result holds regardless of whether or not the economy has access to borrowing and lending. Having access to global capital markets, however, is shown to be equivalent to a reduction in the distributive share of energy resources in production of final output.

Several interesting results emerge when we look at what role the cost of credit, r, plays in determining the optimal investment choice and the economy's net foreign asset position (NFA). First, it is shown that, depending on the relationship between r and the rate of time preference ρ, RIC may be either a borrower or a lender, and in particular, the lending phase may precede the phase of borrowing. Second, a successful R&D outcome causes an improvement in the NFA when r is not too low in relation to ρ but a *deterioration* in the NFA for low enough interest rates. Third, the economy's expected lifetime welfare with access to credit always exceeds the one obtained under financial autarky, regardless of the value of r. Moreover, the welfare with access to credit is U-shaped in r due to the dual role of the latter in the resource and capital markets. Finally, the optimal investment rate responds differently to variations in r depending on whether access to credit is available or not: it is an increasing function of r under financial autarky but a *decreasing* function of r under openness.

The present analysis motivates the desire to invent a substitute for a non-renewable resource by its increasing market price and thus increasing dependence on energy imports. Introducing other motivations for a switch from non-renewable to renewable sources of energy, such as an objective to meet a specific climate-policy target, would enrich the analysis even further. Assuming that the social planner dislikes pollution and the backstop is a clean energy source, there would be an additional incentive to invest in R&D.

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A Appendix

A.1 Transforming a Stochastic Control Problem into a Deterministic Control Problem

In the case of financial autarky the optimization problem is to maximize

$$E_{\tau}\left\{\int_0^{\tau} u(c_t)e^{-\rho t}dt + \int_{\tau}^{\infty} u(\tilde{c}_t)e^{-\rho t}dt\right\}, \tag{21}$$

subject to $c_t = Y_t^n - m_t$ and $\tilde{c}_t = \bar{Y}$, where E_{τ} denotes the expectation operator with respect to the distribution of the arrival date. Given that

$$\mathcal{P}[\tau \in (t, t+dt)|\tau \geqslant t] = q(m_t)dt + o(dt),$$

the elementary probability on the interval $(t, t+dt)$ is given by $q(m_t)e^{-\int_0^t q(m_s)ds}dt$. Then (21) can be rewritten as

$$\int_0^{\infty}\left\{\int_0^t u(c_s)e^{-\rho s}ds + \int_t^{\infty} u(\tilde{c}_s)e^{-\rho s}ds\right\} q(m_t)e^{-\int_0^t q(m_s)ds}dt. \tag{22}$$

Since the consumption rate after the arrival of the backstop is constant at $\bar{Y}$, the last term in the curly braces equals to $u(\bar{Y})\frac{e^{-\rho t}}{\rho}$, and (23) can be written as

$$\int_0^{\infty}\left\{\int_0^t u(c_s)e^{-\rho s}ds\right\} q(m_t)e^{-\int_0^t q(m_s)ds}dt + \frac{u(\bar{Y})}{\rho}\int_0^{\infty} q(m_t)e^{-\rho t-\int_0^t q(m_s)ds}dt. \tag{23}$$

Defining $\mathcal{U}(t) = \int_0^t u(c_s)e^{-\rho s}ds$ and $\mathcal{V}(t) = -e^{-\int_0^t q(m_s)ds}$, we can apply integration by parts to the first term to obtain:

$$\int_0^\infty \left\{ \int_0^t u(c_s)e^{-\rho s}ds \right\} q(m_t)e^{-\int_0^t q(m_s)ds}dt = \tag{24}$$

$$\int_0^\infty \mathcal{U}(t)d\mathcal{V}(t) = \tag{25}$$

$$\mathcal{U}(t)\mathcal{V}(t) - \int_0^\infty \mathcal{V}(t)d\mathcal{U}(t) = \tag{26}$$

$$-\int_0^t u(c_s)e^{-\rho s}ds \left[e^{-\int_0^t q(m_s)ds} \right] + \int_0^\infty e^{-\int_0^t q(m_s)ds}u(c_t)e^{-\rho t}dt. \tag{27}$$

The term $\mathcal{U}(t)\mathcal{V}(t)$ is zero in the limit as t goes to infinity since $\int_0^t u(c_s)e^{-\rho s}ds < \infty$ and $\int_0^\infty q(m_s)ds = \infty$. Then the original objective in (23) becomes

$$\int_0^\infty e^{-\int_0^t q(m_s)ds}u(c_t)e^{-\rho t}dt + \frac{u(\bar{Y})}{\rho}\int_0^\infty q(m_t)e^{-\rho t-\int_0^t q(m_s)ds}dt =$$

$$= \int_0^\infty \left\{ u(c_t) + q(m_t)\frac{u(\bar{Y})}{\rho} \right\} e^{-\rho t-\int_0^t q(m_s)ds}dt. \tag{28}$$

Defining an auxiliary state variable $z_t \equiv \int_0^t q(m_s)ds$ with $\dot{z}_t \equiv \frac{dz}{dt} = q(m_t)$ and $z_0 = 0$, the objective function (28) becomes

$$\int_0^\infty \left\{ u(c_t) + q(m_t)\frac{u(\bar{Y})}{\rho} \right\} e^{-\rho t - z_t}dt, \tag{29}$$

which is used to construct the Hamiltonian (4) in the text.

A.2 Optimal Investment with Open Access to International Lending and Borrowing

The optimal control problem pertaining to phase II is:

$$\max_{\tilde{c}_t} \quad \int_{\tau}^{\infty} u(\tilde{c}_t)e^{-\rho(t-\tau)}dt$$

subject to

$$\dot{a}_t = B^{\alpha}L^{1-\alpha} - \tilde{c}_t + ra_t, \quad \forall t > \tau. \tag{30}$$

The current-value Hamiltonian may be written as

$$H = u(\tilde{c}_t) + \mu_t \left[B^{\alpha}L^{1-\alpha} - \tilde{c}_t + ra_t\right]$$

and the first order conditions

$$\tilde{c}_t : \quad u'(\tilde{c}_t) - \mu_t = 0, \tag{31}$$

$$a_t : \quad \mu_t r = \rho - \dot{\mu}. \tag{32}$$

Differentiating eq. (31) with respect to time and inserting the result in (32) yields the standard Keynes-Ramsey rule

$$\hat{\tilde{c}}_t = \frac{r-\rho}{\theta}, \quad \forall t > \tau$$

and therefore the consumption path

$$\tilde{c}_t = \tilde{c}_\tau e^{\frac{r-\rho}{\theta}(t-\tau)}.$$

Combining this with the budget constraint (30) allows to solve for the consumption rate right after the discovery takes place, $\tilde{c}_\tau$, and for the time path of asset holdings:

$$\tilde{c}_\tau = \left(r - \frac{r-\rho}{\theta}\right)\left(a_\tau + \frac{B^\alpha L^{1-\alpha}}{r}\right), \tag{33}$$

$$a_t = a_\tau e^{\frac{r-\rho}{\theta}(t-\tau)} + \frac{B^\alpha L^{1-\alpha}}{r}\left(e^{\frac{r-\rho}{\theta}(t-\tau)} - 1\right). \tag{34}$$

The the maximized discounted (time-τ) welfare in Phase II is

$$\Phi(a_\tau) = \int_\tau^\infty \frac{\tilde{c}_t^{1-\theta}}{1-\theta} e^{-\rho(t-\tau)} dt = u(\tilde{c}_\tau)(r - \frac{r-\rho}{\theta})^{-1}.$$

The Hamiltonian, associated with the RIC's original optimization problem may be written as

$$H = \{u(c_t) + \lambda(m)\Phi(a_t)\} e^{-\rho t - z_t} + \eta_t(ra_t + R_t^\alpha L^{1-\alpha} - c_t - P_t R_t - m) + \nu_t \lambda(m),$$

where η_t is the co-state variable associated with the constraint (11) and z_t is the auxiliary state variable, such that $\dot{z}_t = \lambda(m)$. The optimality conditions are

$$R_t: \quad \eta_t\left(\frac{\partial F_t}{\partial R_t} - P_t\right) = 0, \tag{35}$$

$$c_t: \quad u(c_t)e^{-\rho t - z_t} - \eta_t = 0, \tag{36}$$

$$m: \quad \lambda'(m)\Phi(a_t)e^{-\rho t - z_t} - \eta_t + \nu_t \lambda'(m) = 0, \tag{37}$$

$$a_t: \quad \lambda(m)\frac{\partial \Phi_t}{\partial a_t} e^{-\rho t - z_t} + r\eta_t = -\dot{\eta}_t, \tag{38}$$

$$z_t: \quad -\Big(u(c_t) + \lambda(m)\Phi(a_t)\Big)e^{-\rho t - z_t} = -\dot{\nu}_t. \tag{39}$$

Combining (36) with (38) yields the Keynes-Ramsey rule under uncertainty:

$$\theta \hat{c}_t = r - \rho - \lambda(m)\left[1 - \frac{u'(\tilde{c}_t)}{u'(c_t)}\right],$$

where I used $u'(\tilde{c}_t) = \frac{\partial \Phi_t}{\partial a_t}$. Isolating ν_t from (37), differentiating with respect to time and inserting the result in (39) yields:

$$u(c_t) = \frac{u''(c_t)\dot{c}_t - u'(c_t)(\rho + \lambda(m))}{\lambda'(m)} + \rho\Phi(a_t) - u'(\tilde{c}_t)\dot{a}_t.$$

The expression in the square brackets can be rewritten in terms of consumption growth rate and then combined with the Keynes-Ramsey rule, so that we get equation (18) in the text:

$$\lambda'(m)\left[\rho\Phi(a_t) - u(c_t) - u'(\tilde{c}_t)\dot{a}_t\right] = u'(c_t)r + \lambda u'(\tilde{c}_t).$$